King Fahd University of Petroleum and Minerals
College of Computer Sciences and Engineering
Information and Computer Science Department

ICS 253: Discrete Structures I
First semester 2013-2014
Major Exam #1, Sunday October 6, 2013
Time: **100** Minutes

Name: ________KEY____

ID#: ______________________

Section: [ ] **01** (Dr. Husni) [ ] **02** (Dr. Abdulaziz) [ ] **03** (Dr. Wasfi)

Instructions:
1. Answer all questions. Show all the steps.
2. Make sure your answers are **clear** and **readable**.
3. The exam is closed book and closed notes. No calculators or any helping aides are allowed. Make sure you turn off your mobile phone and keep it in your pocket.
4. If there is no space on the front of the page, use the back of the page.

Question	Maximum Points	Earned Points	Remarks
1	20		
2	15		
3	10		
4	10		
5	20		
6	10		
7	15		
Total	**100**		

Rules of Inference:

$p \to (p \vee q)$	Addition	$[\neg q \wedge (p \to q)] \to \neg p$	Modus Tollens
$(p \wedge q) \to p$	Simplification	$[(p \to q) \wedge (q \to r)] \to (p \to r)$	Hypothetical syllogism
$[(p) \wedge (q)] \to (p \wedge q)$	Conjunction	$[(p \vee q) \wedge \neg p] \to q$	Disjunctive syllogism
$[p \wedge (p \to q)] \to q$	Modus Ponens	$[(p \vee q) \wedge (\neg p \vee r)] \to (q \vee r)$	Resolution
$\forall x P(x) \to P(a)$ for all a	Universal Instantiation	$\exists x P(x) \to P(a)$ for some a	Existential Instantiation
$P(a)$ for all $a \to \forall x P(x)$	Universal Generalization	$P(a)$ for some $a \to \exists x P(x)$	Existential Generalization

Q1: [20 points] Answer the following questions.

a) [5 points] Re-write the following so that negations appear only before predicates.

$$\neg\exists x\forall y(P(x) \to (\neg Q(x,y) \to R(x)))$$

$\forall x\exists y(P(x) \land \neg Q(x,y) \land \neg R(x))$

b) [6 points] Show that the following is a <u>contradiction</u> using logical identities (Do not use truth tables. Do not assume truth values for propositions).

$$q \land \neg r \land (p \to \neg q) \land (\neg p \to r)$$

$q \land \neg r \land (p \to \neg q) \land (\neg p \to r)$
$\equiv q \land (\neg p \lor \neg q) \land \neg r \land (p \lor r)$
$\equiv (q \land \neg p \lor q \land \neg q) \land (\neg r \land p \lor \neg r \land r)$
$\equiv (q \land \neg p \lor \boldsymbol{F}) \land (\neg r \land p \lor \boldsymbol{F})$
$\equiv q \land \neg r \land \neg p \land p$
$\equiv q \land \neg r \land \boldsymbol{F}$
$\equiv \boldsymbol{F}$

So, it is a contradiction.

c) [6 points] Show that $(\neg p \to (q \to r))$ and $(q \to (p \lor r))$ are logically equivalent without using truth tables.

$(\neg p \to (q \to r))$
$\equiv (p \lor (\neg q \lor r))$
$\equiv (\neg q \lor p \lor r)$
$\equiv (q \to p \lor r)$

So, they are logically equivalent.

d) [3 points] State the inverse, contrapositive and converse of the following implication.

"If x is even then x is not prime."

Inverse: "If x is odd then x is a prime."

Contrapositive: "If x is a prime then x is odd."

Converse: "If x is not a prime then x is even."

Q2: [15 points] Consider the following system specifications:

1) The software will not be installed if it is not purchased.
2) If the software is purchased and there is enough space on the hard disk, the software will be installed.
3) The report will be printed whenever the software is installed.
4) The software is either not purchased or it is installed but cannot be installed and not purchased at the same time.
5) The report is not printed.

a) [9 points] Translate the above specifications to propositional logic using the following propositions:

p: The software will be installed.
q: The software is purchased.
r: There is enough space on the hard drive.
s: The report will be printed.

1) $\neg q \rightarrow \neg p$

2) $q \wedge r \rightarrow p$

3) $p \rightarrow s$

4) $\neg q \oplus p$

5) $\neg s$

b) [6 points] Determine whether the given system specifications are consistent. Clearly justify your answer using logic.

From (5), $s \equiv \boldsymbol{F}$.
From (3), $p \equiv \boldsymbol{F}$.
From (4), $q \equiv \boldsymbol{F}$.
(1) is satisfied.
(2) is satisfied since the conclusion is False, regardless of the value of r.

So, the system specifications are consistent.

Q3: [10 points] Let $P(x)$ be a predicate where the domain of x is the set $\{2, 3, 5, 7\}$. Write the following in simplest form without quantifiers using logical connectives and negation.

a) [5 points] $\exists x((x > 3) \rightarrow P(x))$

$$
\begin{aligned}
&\exists x\big((x > 3) \rightarrow P(x)\big)\\
&\equiv ((2 > 3) \rightarrow P(2)) \vee ((3 > 3) \rightarrow P(3)) \vee ((5 > 3) \rightarrow P(5)) \vee ((7 > 3) \rightarrow P(7))\\
&\equiv (\boldsymbol{F} \rightarrow P(2)) \vee (\boldsymbol{F} \rightarrow P(3)) \vee (\boldsymbol{T} \rightarrow P(5)) \vee (\boldsymbol{T} \rightarrow P(7))\\
&\equiv \boldsymbol{T} \vee \boldsymbol{T} \vee P(5) \vee P(7)\\
&\equiv \boldsymbol{T}
\end{aligned}
$$

b) [5 points] $\forall x((x = 5) \oplus \neg P(x))$

$$
\begin{aligned}
&\forall x((x = 5) \oplus \neg P(x))\\
&\equiv ((2 = 5) \oplus \neg P(2)) \wedge ((3 = 5) \oplus \neg P(3)) \wedge ((5 = 5) \oplus \neg P(5)) \wedge ((7 = 5) \oplus \neg P(7))\\
&\equiv (\boldsymbol{F} \oplus \neg P(2)) \wedge (\boldsymbol{F} \oplus \neg P(3)) \wedge (\boldsymbol{T} \oplus \neg P(5)) \wedge (\boldsymbol{F} \oplus \neg P(7))\\
&\equiv \neg P(2) \wedge \neg P(3) \wedge P(5) \wedge \neg P(7)
\end{aligned}
$$

Q4: [10 points] Determine whether the following arguments are valid or invalid. Justify your answers clearly using the rules of inference.

a) [5 points] "All those who eat apples have strong teeth." "All those who don't eat apples are unhealthy." "Saleem hasn't strong teeth." Therefore "Saleem is unhealthy."

$P(x)$: x eats apples, $Q(x)$: x has strong teeth, $R(x)$: x is healthy.
Argument: $[\forall x(P(x) \rightarrow Q(x)) \wedge \forall x(\neg P(x) \rightarrow \neg R(x)) \wedge \neg Q(\text{Saleem})] \rightarrow \neg R(\text{Saleem})$

By Universal Instantiation, $P(\text{Saleem}) \rightarrow Q(\text{Saleem})$ and $\neg P(\text{Saleem}) \rightarrow R(\text{Saleem})$.
By Modus Tollens, $\neg P(\text{Saleem})$.
By Modus Ponens, $\neg R(\text{Saleem})$. The argument is **valid**.

b) [5 points] "It is not raining or Hussain has his coat." "Hussain does not have his coat or he does not get wet." "It is raining or Hussain does not get wet." Therefore "Hussain does not get wet."

r: It is raining, c: Hussain has his coat, w: Husain gets wet.
Argument: $(\neg r \vee c) \wedge (\neg c \vee \neg w) \wedge (r \rightarrow \neg w) \rightarrow \neg w$

By Resolution, $(\neg r \vee c) \wedge (\neg c \vee \neg w) \rightarrow (\neg r \vee \neg w)$.
By Resolution, $(\neg r \vee \neg w) \wedge (r \rightarrow \neg w) \rightarrow \neg w$. The argument is **valid.**

Q5: [20 points] Let $F(x,y)$ be the statement "x and y are friends" and $V(x,c)$ be the statement "x has visited c", where x and y are humans and c is a country.
Use quantifiers to represent each of the following statements:

a) [5 points] A friend of Ahmad has visited the US.

$$\exists x(F(\text{Ahmed}, x) \land V(x, \text{US}))$$

b) [5 points] Ahmad has visited all the countries that Ali has visited.

$$\forall x(V(\text{Ali}, x) \rightarrow V(\text{Ahmad}, x))$$

c) [5 points] A friend of Ahmad has not visited any of the countries that Ahmad has visited.

$$\exists x \forall y \left(F(\text{Ahmed}, x) \land \left(V(\text{Ahmad}, y) \rightarrow \neg V(x, y)\right)\right)$$

d) [5 points] At least two people have visited the US, though not everybody has visited it.

$$\exists x \exists y \exists z\big((x \neq y) \land V(x, \text{US}) \land V(y, \text{US}) \land \neg V(z, \text{US})\big)$$

Q6: [10 points] Recall the island of the knights and knaves, where knights always tell the truth and knaves always lie. On that island, you encounter three people called A, B and C. Determine, if possible, the types of A, B and C if they said the following to you:

A said: "B and C are knights."
B said nothing.
C said: "At least one of A and B is a knave."

Let $K(x)$ represent the statement "x is a knight."
From what A said, we get $K(A) \leftrightarrow K(B) \land K(C)$.
From what C said, we get $K(C) \leftrightarrow \neg K(A) \lor \neg K(B)$

Case 1: If A is a knight, then both B and C are knights, but this implies that C is a knave as he is not telling the truth, so A cannot be a knight.

Case 2: If A is a knave, then either B or C (or both) are knaves.
If C is a knave, then both A and B have to be knights, but that is false since A is a knave.
If C is a knight, then either A or B are knaves, which is true since A is a knave.
As for B, he has to be a knave because, if he was a knight, then A is telling the truth.

So, we conclude that **A is knave, B is a knave, and C is a knight**.

Q7: [15 points] Use the rules of inference to show that:

(1) $\forall x\left(\left(P(x) \vee Q(x)\right) \to R(x)\right)$,

(2) $\forall x\left(R(x) \to S(x)\right)$,

(3) $\exists x\left(\neg S(x) \wedge T(x)\right)$

imply $\exists x \neg P(x)$.

[1] $\neg S(a) \wedge T(a)$ for some a,	from (3)	by Existential Instantiation.
[2] $\neg S(a)$,	from [1]	by Simplification.
[3] $R(a) \to S(a)$,	from (2)	by Universal Instantiation.
[4] $\neg R(a)$,	from [2], [3]	by Modus Tollens.
[5] $\left(P(a) \vee Q(a)\right) \to R(a)$,	from (1)	by Universal Instantiation.
[6] $\neg P(a) \wedge \neg Q(a)$,	from [4], [5]	by Modus Tollens.
[7] $\neg P(a)$,	from [6]	by Simplification.
[8] $\exists x \neg P(x)$,	from [7]	by Existential Generalization.